Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

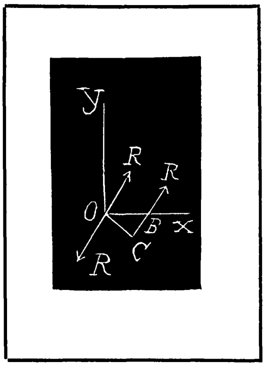
JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

TO FIND THE EQUATION TO THE STRAIGHT LINE WHICH IS THE DIRECTION OF THE RESULTANT OF A SYSTEM OF FORCES ACTING IN ONE PLANE.

By G. B. M. ZERR, A. M., Ph. D.

This article is not presented with the idea of developing something new, but for the purpose of collecting the old together into a form convenient for the solution of such problems as the example given below.

All the forces in one plane can be resolved into the resultant R acting through the origin O and a couple H.



Let G be the moment of the couple, θ the angle R makes with the axis of abscissas, P any one of the forces, β the angle P makes with the axis of abscissas. Also, let $X=P\cos\beta$, $Y=P\sin\beta$.

Then $G=\Sigma(Yx-Xy)$, $\tan\theta=\Sigma Y/\Sigma X$. Transform G into a couple having its two forces each$=R$: the one acting at O directly opposite to R, and, hence, destroying it; the other, at a distance $CO=G/R$ from the origin.

Then $OB=OC\operatorname{cosec}\theta=\dfrac{G}{R}.\dfrac{R}{\Sigma Y}=\dfrac{G}{\Sigma Y}$.

The equation to the line required is, therefore, $y=\tan\theta(x-OB)=(\Sigma Y/\Sigma X)(x-G/\Sigma Y)$.

$\therefore y\Sigma X=x\Sigma Y-G$.........(1).

Problem. Each element of the arc of an elliptic quadrant is acted on by a force in the normal proportional to the ordinate of that point. Find the equation to the straight line along which the resultant acts.

Let $(m,\ n)$ be the coördinates of the point.

Then $y-n=\dfrac{a^2n}{b^2m}(x-m)$ is the normal.

$\therefore \tan\beta=\dfrac{a^2n}{b^2m}$, $P=Cn$, suppose.

$$\therefore Cn\cos\beta=\frac{Cb^2mn}{\sqrt{(b^4m^2+a^4n^2)}}=\frac{Cb^2mn}{a\sqrt{[b^4+(a^2-b^2)n^2]}}=X,$$

$$Cn\sin\beta=\frac{Ca^2n^2}{\sqrt{(b^4m^2+a^4n^2)}}=\frac{Ca^2n^2}{a\sqrt{[b^4+(a^2-b^2)n^2]}}=Y,$$

$$\frac{C(a^2-b^2)n^2m}{a\sqrt{[b^4+(a^2-b^2)n^2]}}=Yn-Xm,\quad \Sigma X=\int_0^b Xds\Big/\int_0^b dn=\frac{1}{b}\int_0^b Xds.$$

Similarly, $\Sigma Y = \frac{1}{b}\int_0^b Y ds$, $G = \Sigma(Yx - Xy) = \frac{1}{b}\int_0^b (Yx - Xy) ds$,

$$ds = \frac{\sqrt{[b^4 + ca^2 - b^2)n^2]dn}}{b\sqrt{(b^2 - n^2)}}, \quad m = \frac{a}{b}\sqrt{(b^2 - n^2)}.$$

$$\therefore \Sigma X = \frac{C}{b}\int_0^b ndn = \tfrac{1}{2}bC, \quad \Sigma Y = \frac{Ca}{b^2}\int_0^b \frac{n^2 dn}{\sqrt{(b - n^2)}} = \tfrac{1}{4}\pi a C,$$

$$G = \frac{C(a^2 - b^2)}{b^3}\int_0^b n^2 dn = \frac{C}{3}(a^2 - b^2).$$

Substituting in (1) we get, $\frac{1}{2}bCy = \frac{1}{4}\pi axC - \frac{C}{3}(a^2 - b^2)$.

$\therefore 6by - 3\pi ax + 4a^2 - 4b^2 = 0$.

We generalize as follows: Suppose each element of the arc of the curve $y = f(x)$, between the limits $y = 0$ and $y = k$, $(x = h)$, is acted on by a force in the normal proportional to the ordinate of the point; to find the equation to the line of action of the resultant.

Let (m, n) be the point, and let $\sqrt{\{1 + [f'(m)]^2\}} = \mathrm{D}$.

Then $\tan\beta = -\frac{dm}{dn} = -\frac{1}{f'(m)}$.

$$\therefore X = Cn\cos\beta = \frac{Cnf'(m)}{\mathrm{D}}, \quad Y = Cn\sin\beta = -\frac{Cn}{\mathrm{D}},$$

$$(Ym - Xn) = -\frac{Cn[m + nf'(m)]}{\mathrm{D}}, \quad ds = \frac{\mathrm{D}dn}{f'(m)} = \mathrm{D}dm.$$

$$\therefore \Sigma X = \frac{C}{k}\int_0^k ndn = \tfrac{1}{2}Ck, \quad \Sigma Y = -\frac{C}{k}\int_0^k \frac{ndn}{f'(m)}, \quad G = \frac{C}{k}\int_0^k \frac{n[m + nf'(m)]dn}{f'(m)}.$$

$$\therefore G = \frac{C}{k}\int_0^k \frac{nm\,dn}{f'(m)} + \tfrac{1}{3}Ck^2.$$

$$\therefore 3ky + \frac{6x}{k}\int_0^k \frac{ndn}{f'(m)} = \frac{6}{k}\int_0^k \frac{nm\,dn}{f'(m)} + 2k^2,$$

is the equation to the required line.

Consider the special example, $f(x) = 2\sqrt{ax}$, whence $y^2 = 4ax$. Let the arc extend from the vertex to one extremity of the latus-rectum.

$\therefore k = 2a, \quad m = n^2/4a, \quad f'(m) = \sqrt{(a/m)} = 2a/n.$

$$\therefore \frac{6}{k}\int_0^k \frac{ndn}{f'(m)} = \frac{6}{2a}\int_0^{2a} \frac{n^2 dn}{2a} = 4a, \quad \frac{6}{k}\int_0^k \frac{nm\,dn}{f'(m)} = \frac{6}{2a}\int_0^{2a} \frac{n^4\,dn}{8a^2} = \frac{12a^2}{5}.$$

$\therefore 6ay + 4ax = \frac{12}{5}a^2 + 8a^2 = \frac{52}{5}a^2$.

$\therefore 15y + 10x = 26a$, is the required equation.